Energy 2

① $m = 0.50\text{ mg} \xrightarrow{\div 1000} 5 \times 10^{-4}\text{ g} \xrightarrow{\div 1000} 5 \times 10^{-7}\text{ kg}$

$v = 30\text{ cm/s} \xrightarrow{\div 100} 0.30\text{ m/s}$

$$E_K = \frac{1}{2} m v^2 = \frac{1}{2}(5 \times 10^{-7})(0.30)^2 = \boxed{2.25 \times 10^{-8}\text{ J}}$$

② $E_K = \frac{1}{2} m v^2$

$$v = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2(4.64 \times 10^{-19})}{(1.99 \times 10^{-26})}}$$

$$v = \boxed{6828.8\text{ m/s}}$$

③ $v = \sqrt{\frac{2E_K}{m}}$ (from #2)

If E_K is doubled, replace E_K by $2E_K$:

$$v' = \sqrt{\frac{2(2E_K)}{m}}$$

$$= \sqrt{2} \cdot \sqrt{\frac{2E_K}{m}}$$

$$v' = \sqrt{2} \cdot v$$

The speed increased by a factor of $\boxed{\sqrt{2}}$.

④ $E_K = \frac{1}{2} m v^2$

If v is doubled, replace v by $2v$:

$$E_K' = \frac{1}{2} m (2v)^2$$
$$= 4 \cdot \frac{1}{2} m v^2$$

$$E_K' = 4 E_K$$

The kinetic energy increased by a factor of $\boxed{4.}$

⑤ $$W = \Delta E_K$$
$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$
$$= \frac{1}{2} (9.11 \times 10^{-31}) (5 \times 10^6)^2 - 0$$
$$W = \boxed{1.14 \times 10^{-17} \text{ J}}$$

⑥ $$W = \Delta E_K$$
$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$
$$= \frac{1}{2} (1000) (0)^2 - \frac{1}{2} (1000) (27.\bar{7})^2$$
$$W = \boxed{-385\ 802 \text{ J}}$$

⑦ $W = F \cdot d$

$W = \Delta E_K$

$\therefore \quad F \cdot d = \Delta E_K$

$$F(0.15) = \tfrac{1}{2}(0.14)(0)^2 - \tfrac{1}{2}(0.14)(30)^2$$

$$F = \boxed{-420\ N}$$

⑧ $F d = \Delta E_K$

↑ braking distance

$$F d = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2$$

$$F d = -\tfrac{1}{2} m v_i^2 \qquad (\text{since } v_f = 0)$$

$$d = -\tfrac{1}{2} \frac{m v_i^2}{F}$$

Since v is increased by 50%, replace v_i by $1.5 v_i$:

$$d' = -\frac{m(1.5 v_i)^2}{2F}$$

$$= 2.25 \left(-\frac{m v_i^2}{2F} \right)$$

$$d' = 2.25\, d$$

The braking distance increases by a factor of $\boxed{2.25}$.

(9) a) $W = \Delta E_K$

$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$= \frac{1}{2}(80)(0)^2 - \frac{1}{2}(80)(50)^2$

$W = \boxed{-100\ 000\ \text{J}}$

b) $W = Fd$

$-100\ 000 = F(1.1)$

$F = \boxed{-90\ 909.1\ \text{N}}$

c) $W = -\vec{F}_f \cdot d$

$= -\vec{F}_g \cdot d$ (at terminal velocity, $\vec{F}_f = \vec{F}_g$)

$= -mg\,d$

$= -(80)(9.8)(370)$

$W = \boxed{-290\ 080\ \text{J}}$

10 a) $\Sigma F = T - F_g$

$$ma = T - mg$$

$$T = ma + mg$$

$$= m(0.15g) + mg$$

$$= 1.15\,mg$$

$$T = (1.15)(180)(9.8)$$

$$T = \boxed{2028.6 \text{ N}}$$

b) $\Sigma W = \Sigma F \cdot d$

$$= ma \cdot d$$

$$= (180)(0.15)(9.8)(23)$$

$$\Sigma W = \boxed{6085.8 \text{ J}}$$

c) $W_T = T \cdot d$

$$= (2028.6)(23)$$

$$W_T = \boxed{46\,657.8 \text{ J}}$$

d) $W_g = F_g \cdot d$

$$= (180)(-9.8)(23)$$

$$W_g = \boxed{-40\,572 \text{ J}}$$

(10) e) $\Sigma W = \Delta E_k$

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$6085.8 = \frac{1}{2}(180)v_f^2 - 0$$

$$v_f^2 = 67.62$$

$$v_f = \boxed{8.22 \text{ m/s}}$$

(11) This question is very challenging and would <u>never</u> appear on anything worth marks.

Car 1	Car 2	
m_a	m_b	
E_{Ka}	E_{Kb}	at original speeds
v_a	v_b	
E_{Ka}'	E_{Kb}'	at new speeds
v_a'	v_b'	

<u>Equations</u>:

$$m_a = 2m_b$$

$$E_{Ka} = \frac{1}{2}E_{Kb}$$

$$v_a' = v_a + 5$$

$$v_b' = v_b + 5$$

$$E_{Ka}' = E_{Kb}'$$

This must be solved algebraically using a system of equations. See next page.

(11) **Part 1**

$E_{Ka} = \frac{1}{2} E_{Kb}$

$\frac{1}{2} m_a v_a^2 = \frac{1}{2}\left(\frac{1}{2} m_b v_b^2\right)$

$\frac{1}{2}(2m_b) v_a^2 = \frac{1}{4} m_b v_b^2$ (sub $m_a = 2m_b$)

$\cancel{m_b} v_a^2 = \frac{1}{4} \cancel{m_b} v_b^2$

$v_a^2 = \frac{1}{4} v_b^2$ take square root of both sides

$v_a = \frac{1}{2} v_b$

Part 2

$E_{Ka}' = E_{Kb}'$

$\cancel{\frac{1}{2}} m_a v_a'^2 = \cancel{\frac{1}{2}} m_b v_b'^2$

$m_a (v_a + 5)^2 = m_b (v_b + 5)^2$ (sub $v_a' = v_a + 5$, $v_b' = v_b + 5$)

$2\cancel{m_b}(v_a + 5)^2 = \cancel{m_b}(v_b + 5)^2$ (sub $m_a = 2m_b$)

$2(v_a + 5)^2 = (v_b + 5)^2$

$2\left(\frac{1}{2} v_b + 5\right)^2 = (v_b + 5)^2$ (sub $v_a = \frac{1}{2} v_b$)

$2\left(\frac{1}{4} v_b^2 + 5 v_b + 25\right) = v_b^2 + 10 v_b + 25$

$\frac{1}{2} v_b^2 + \cancel{10 v_b} + 50 = v_b^2 + \cancel{10 v_b} + 25$

(continued on next page)

⑪ continued...

$$\frac{1}{2} v_b^2 + 50 = v_b^2 + 25$$

$$50 - 25 = v_b^2 - \frac{1}{2} v_b^2$$

$$25 = \frac{1}{2} v_b^2$$

$$50 = v_b^2$$

$$v_b = 7.07 \text{ m/s}$$

$$v_a = \frac{1}{2} v_b$$

$$= \frac{1}{2}(7.07)$$

$$v_a = 3.54 \text{ m/s}$$

$$\boxed{\begin{aligned} v_a &= 3.54 \text{ m/s} \\ v_b &= 7.07 \text{ m/s} \end{aligned}}$$